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## EVOLUTION OF RADIATION SPECTRA OF RELATIVISTIC POSITRONS AT (100) AND (111) CHANNELING IN Si CHANGING OF ANGLE INCIDENCE AND PARTICLE ENERGY

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Evolution of radiation spectral intensity has been investigated at (111) positron channeling in thin crystals Si at in energy of positrons increase from 100 up to 300 MeV and change of the angle of incidence. It was revealed that spectra have more complicate structure and a full output of radiation is a few times higher than at (100) or (110) channeling, that is explained by presence of additional group of positrons on transverse section energy.

### Introduction

Theory of relativistic charged particle radiation was detailed developed in monographs [1–4]. Characteristics of electron and positron radiation at planar channeling, radiation at channeling (CR), along planes (100) and (110) of diamond, silicon and germanium crystals are more detailed studied in the work [1] by approximation of potential of a single crystal plane by a simple function of the type «inverse parabola». However, analytical treatment is complicated at channeling along so-called double planes (111). It is connected to the following reasons:

- potential of double plane system (111) is complicated (Fig. 1, a);
- analytical solution of equation of motion is impossible to be found in such potential.

Trajectory of a particle in a crystal may be determined by a model of binary collisions [4] or by numerical solution of equation of motions. In the paper the second approach is used within which:

- potential of double plane system (111) is calculated on the basis of the model Kh. Chouffani.
- trajectories and velocities of positrons are obtained by numerical integration of equation of motion neglecting dechanneling (thin crystal,  $l < l_d = \alpha L_{rad}$ ,  $U_0 \varepsilon / \pi m^2 c^4$ , here  $\alpha = 1/137$ ,  $U_0$  is the depth of potential well;  $\varepsilon$  is the positron energy,  $L_{rad}$  is the radiation length,  $m$  is the particle mass,  $c$  is the light speed). In the paper the range of positron energies was selected according to parameters of electron and positron beams LNF (Frascati).
- Fourier velocity components necessary for computing spectral-angular distribution of radiation intensity are numerically determined.

### Theoretical analysis

Equation of motion of relativistic positron in potential (Fig. 1, a) for transverse direction (at approximation  $v_{\perp} \ll c$ ;  $v_{\perp}/c \ll 1/\gamma$ ) has the form:

$$\gamma m \ddot{x} = F = -\frac{\partial U(x)}{\partial x}, \quad \gamma = \frac{1}{\sqrt{1 - v_{\parallel}^2/c^2}}, \quad (1)$$

$v_{\parallel}$  is the average rate of positron motion.

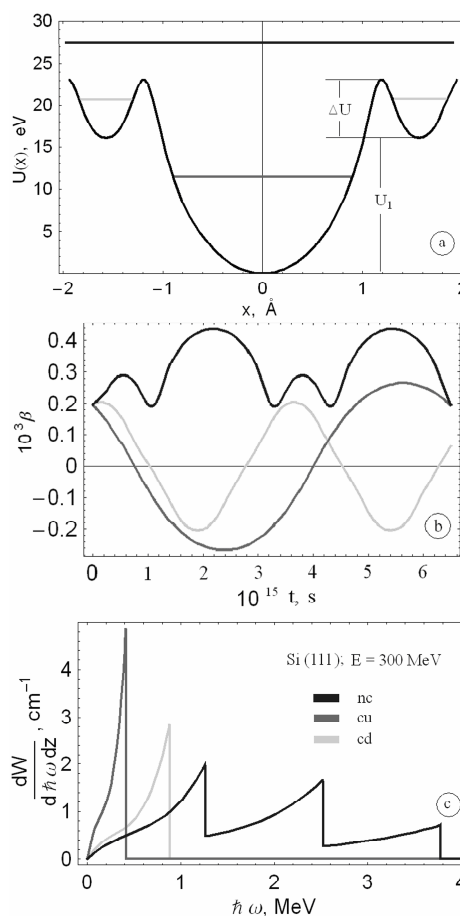


Fig. 1. a) Positron potential energy in double plane system (111) Si and three values of transverse energy corresponding to three motion types; b) dependence of transverse velocity  $\beta_{\perp} = \dot{x}/c$  on time for three specified values of transverse energies; c) CR spectra from each of three trajectories

Initial conditions are: a point of transverse into crystal  $x(0) = x_0$  and transverse momentum  $p_{\perp}(0) = p_{\parallel} \theta_0$  determining integral of equation motion (1), — a so-called transverse energy:

$$\varepsilon_{\perp} = U(x) + \frac{p_{\perp}^2}{2\gamma m} = U(x_0) + \frac{p_{\parallel}^2 \theta_0^2}{2\gamma m}. \quad (2)$$

Here  $\theta_0$  is the angle of incidence of positron relative to channeling plane. Depending on the ratio between  $\theta_0$  and critical angle for channeling  $\theta_c = \sqrt{2U_0/\varepsilon}$  as well as on a height

of internal potential barrier  $\Delta U = U_l - U_0$  (Fig. 1, *a*) at double plane moving in potential instead of two (as at (100) or (110)) three types of solution of equation (1) are possible for transverse coordinate and velocity. They describe the motion of channeled (underbarrier) or quasichanneled (above-barrier) positrons (Fig. 1, *b*). Classification of solution types is given in the Table where *cd* is the motion in a channel (bounded motion or channeling near two planes), *cu* is the motion in a channel (bounded motion or channeling near one plane) and *nc* is the above-barrier motion in periodic field formed by a periodic sequence of potentials (Fig. 1, *a*).

**Table.** Classification of equation of motion (1) solutions in a task about (111) channeling of relativistic positrons in a crystal Si

Trajectory type	Crosscut energy	Period
<i>cd</i>	$0 \leq \varepsilon_{\perp} \leq U_0$	$T_{cd}$
<i>cu</i>	$U_l \leq \varepsilon_{\perp} \leq U_0$	$T_{cu}$
<i>nc</i>	$U_0 \leq \varepsilon_{\perp}$	$T_{nc}$

According to three types of motion three groups of periods of motion – two ones for channeled positrons  $T_{cd}$ ,  $T_{cu}$  and one is for above-barrier positrons  $T_{nc}$  occur in the task.

Thus, at transition from bounded under-barrier motion of the type *cd* to a bounded under-barrier motion of the type *cu* the period of motion decreases in more than twice therefore, a typical radiation frequency of sub-barrier positrons of the type *cd* should be approximately twice as much. Then, at transition from bounded under-barrier motion of the type *cu* to the above-barrier motion the period of motion increases again approximately twice, therefore, a typical radiation frequency of above-barrier positrons should be approximately twice as much that is comparable to the frequency of positron radiation in motion at trajectories of the type *cu*.

If the angle of positron incidence into the crystal relative to a plane  $\theta_0 \neq 0$  then positrons for which  $0 \leq \varepsilon_{\perp} \leq U_0$  enter the channel and move in a mode bounded with one or two planes. Positrons for which  $U_0 \leq \varepsilon_{\perp}$  move out of channel crossing sequentially (in the transverse direction) crystal planes placed periodically. Radiation of these three types of particles in general case in thin crystal can not be separately – in the experiment beam positrons possess different angles and points of incidence  $x_0$  into crystal and therefore obtain different values of potential energy  $U(x_0)$ , therefore, different values of transverse energy  $\varepsilon_{\perp}$ .

Solution (1) was carried out numerically. All characteristics of radiation occurring at quasi-periodic motion of these particles may be determined by the found out positron velocities using proper formulae of classical electrodynamics. Let us use the formula for calculation CR of single positron in the form [1]:

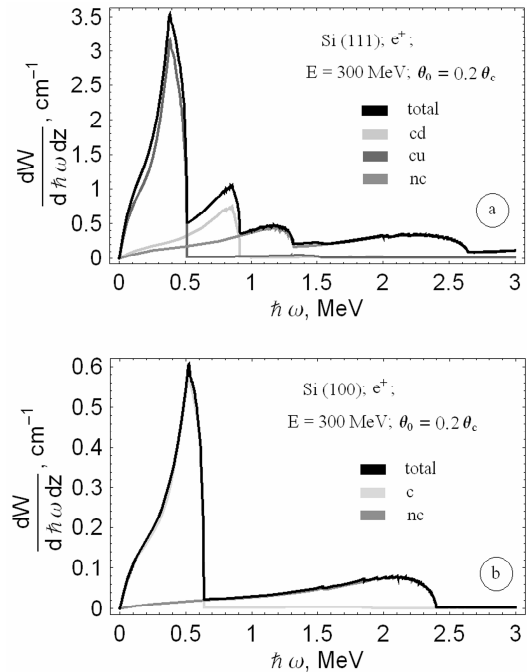
$$\begin{aligned} \frac{dW}{d\omega dz} &= \frac{e^2 \omega}{c^4 T^2} \sum_{l=1}^{\infty} \Theta[1 - \eta_l] \cdot [\eta_l^2 - \eta_l + \frac{1}{2}] \cdot |\dot{x}_{\omega}|^2; \\ \eta_l^c &= \frac{T\omega}{2\pi\gamma^2(2l-1)}; \quad \eta_l^{nc} = \frac{T\omega}{2\pi\gamma^2 l}; \\ \tilde{\omega} &= \frac{2\pi l}{T}; \quad \dot{x}_{\omega} = \int_{-T/2}^{T/2} \dot{x} e^{i\tilde{\omega}t} dt, \end{aligned} \quad (3)$$

where  $W$ ,  $\omega$  is the energy and frequency of photons;  $\Theta[1 - \eta_l]$  is the step function of Heaviside;  $T = \{T_{cd}, T_{cu}, T_{nc}\}$  is the period of positron motion,  $e$  is the electron charge,  $z$  is the crystal thickness. CR spectra for typical trajectories (characterized by transverse energy (2) and velocity, Fig. 1 *a, b*) calculated by the formula (3) are given in Fig. 1, *c*.

#### Evolution of IR spectra at positron transverse angle change

Fig. 2–4 show evolutions of spectra (100) and (111) CR increasing positron incidence in angle into crystal  $\theta_0$  (orientation dependence).

For the case (111) CR spectrum is formed by summing contributions of three groups of particles according to the accepted classification (Table). CR spectrum of each particle consists, in its turn, of a sum of harmonics (3). Difference of harmonic typical frequencies for each group results in overlapping of radiation spectra of particles of three groups. The degree of spectra overlapping depends on a number of particles in each group that is a incidence in angle  $\theta_0$ . Our calculations showed that selecting a incidence in angle  $\theta_0$  may result in more distinct separation of contributions of three groups of positrons into summary IR spectrum (111). IR spectrum (curves *total*) averaged by the points of positron incidence into crystal for the angle of incidence  $\theta_0 = 0, 2\theta_c$  relative to the plane (111) and (100) Si,  $E = 300$  MeV is shown in Fig. 2; it is seen there that CR spectrum (111) is divided into 3 humps and CR spectrum (100) into two.



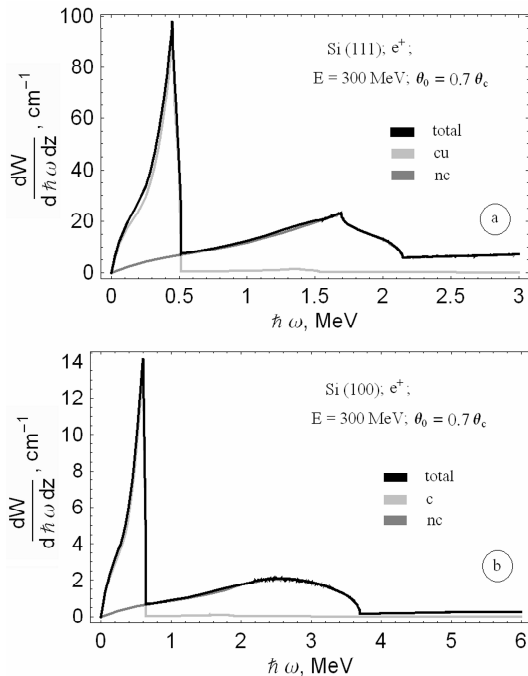
**Fig. 2.** Spectrum of CR positrons at incidence in angle into crystal Si  $\theta_0 = 0, 2\theta_c$  relative to the planes: a) (111); b) (100)

Let us note that motion type *cu* exists for the range of the angle of incidence  $\theta_0 = 0 \dots \theta_{sc}$ :

$$\theta_{sc} = \sqrt{\frac{2\Delta U}{\varepsilon}} = \sqrt{\frac{\Delta U}{U_0}} \theta_c, \quad (4)$$

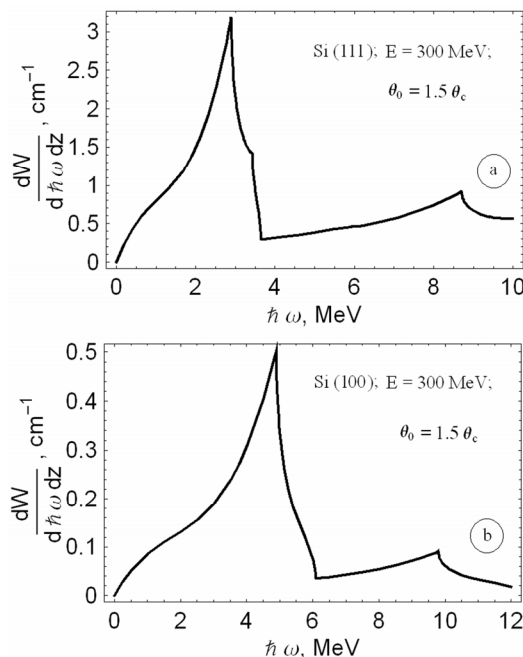
here subcritical angle  $\theta_{sc}$  is introduced, in our case:  $\theta_{sc}=0,552\theta_c$ .

CR spectra for the angle of incidence  $\theta_0=0,7\theta_c$  relative to the plane (111) and (100) Si,  $E=300$  MeV are shown in Fig. 3. At angle of incidence  $\theta_0=\theta_{sc}... \theta_c$  summary spectra form two groups of positrons by transverse energies.



**Fig. 3.** Spectra of CR positrons at transverse in angle into crystal Si  $\theta_0=0,7\theta_c$  relative to planes: a) (111); b) (100)

IR spectra for the angle of incidence  $\theta_0=1,5\theta_c$  relative to the planes (111) and (100) Si,  $E=300$  MeV are shown in Fig. 4. For the angle of incidence  $\theta_0>\theta_c$  all positrons move over the barrier.

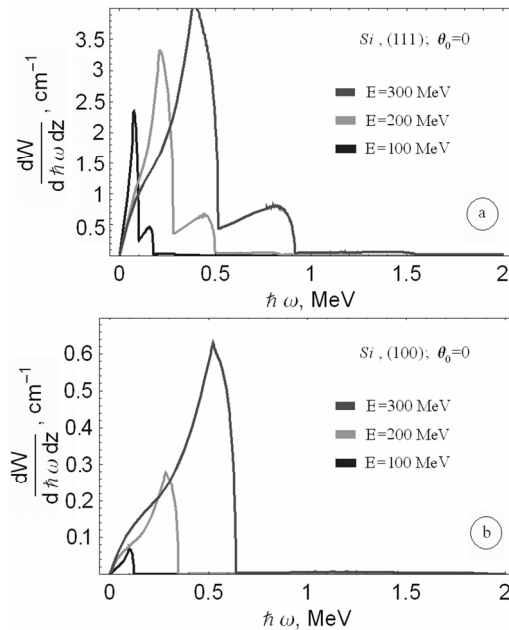


**Fig. 4.** Spectrum of CR positrons at incidence in angle into crystal Si  $\theta_0=1,5\theta_c$  relative to the planes: a) (111); b) (100)

The principle differences of spectra, Fig. 4, are connected with difference of parameters  $d$ ,  $U_0$  and form (111) and (100) of potentials.

#### Evolution of CR spectra at positron energy increase

Spectral distribution of CR intensity averaged by points of positron incidence into crystal per a unit of crystal length for a transverse in angle  $\theta_0=0$  at channeling along the planes (111) and (100) is given in Fig. 5.



**Fig. 5.** Evolution of CR positron spectra at energy change from 100 to 300 MeV at a fixed incidence in angle into crystal Si  $\theta_0=0$  relative to the planes: a) (111); b) (100)

Spectra are given in absolute units therefore both intensity growth at positron energy increase 100...300 MeV and typical maximums shift ( $\gamma^{3/2}$  dependence) into firmer part of radiation spectrum may be traced. At angle  $\theta_0=0$  all particles fall into channel as  $\varepsilon_{\perp}<0$ . Presence of additional group of positrons by transverse energy for (111) potential of double planes results in sharp difference of spectrum (Fig. 5, a, b) and consists in appearance of spectrum two-hump structure.

#### Conclusion

Evolution of CR spectra at increase of energy and positron incidence in angle to (111) channeling planes in a thin crystal Si (neglecting dechanneling) was firstly detailed studied by numerical calculations. It is shown that at (111) channeling the form of radiation spectrum is more complex in comparison with (100), (110) CR spectra that is explained by a presence of additional group of positrons by a transverse energy. Let us note that complete yield of photons (or integral by photon energy) for (111) channeling is rather high than for (100) channeling (for example, for Si at  $\theta_0=0,2\theta_c$  and  $E=300$  MeV –  $Y(111)=1,94$  MeV/cm and  $Y(100)=0,27$  MeV/cm) that may be of interest from the point of view of developing the source of quasi-monochromatic photons for photonuclear physics.

Experimental detection of rated peculiarities of IR spectra at (111) channeling in a thin crystal Si is possible at initial angular divergence of positron beam  $\Delta\theta_0 < \theta_{sc}$ . Angular distribution and polarization as well as

account of dechanneling and electron comparison with IR are examined in a separate work.

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## GAMMA SPECTROMETRIC METHOD TO CONTROL ACTIVITY AND NUCLIDE COMPOSITION OF GASEOUS RADIOACTIVE WASTE FORMED AT OPERATION OF NUCLEAR POWER PLANTS

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*Gamma spectrometric method was developed to monitor continuously and on line radioactivity and nuclide composition of inert radioactive gases, radioactive aerosols and iodine in gas aerosol emissions from power reactor facilities. This method is based on continuous representative sampling of gas aerosol samples and quasi-continuous automated recording of nuclide composition and radioactive material emission rate. Low detectable level of the method is about 0,1 Bq/m<sup>3</sup>, highest detectable level for noble gases (Ar-41, isotopes Xe and Kr) is about 10<sup>6</sup> Bq/m<sup>3</sup>.*

## Introduction

Increase of quality of controlling radioactive gas aerosol emissions is one of the key directions of radiation safety at operation of power reactor facilities (PRF) [1].

At PRF operation the main components of gaseous radioactive wastes (RAW) are: inert radioactive gases (Ar-41, isotopes Xe and Kr), radioactive aerosols and gaseous iodine [2].

Application of radiometric methods for continuous control of gas aerosol emissions of PRF allows determining just integral activity of gas released in atmosphere. In conditions of variable nuclide composition of gas aerosol emissions the radiometric method does not allow estimating contribution of individual radionuclides into total activity [3]. It is necessary to determine a contribution of individual radionuclides for estimating the consequences of emission and reporting towards control organizations. Radioisotope composition of emission may be analyzed by semiconductor gamma spectrometers [4].

The feature of using laboratory spectrometers for controlling emissions is the fact that measurements are not carried out directly at gas main lines but by sampling. Gas aerosol sampling is carried out during a long period of time (a day, as a rule). After that a sample (aerosol filter, gas capacity, coal column etc.) is taken by operator from a sampling line and put to a semiconductor detector of gamma spectrometer for measuring.

Gamma spectrometric analysis allows determining activity and nuclide composition of a sample as well as integral quantitative characteristics of gas aerosol emission in general. However, application of laboratory gamma spectrometers for controlling activity and nuclide composition of radioactive emissions of PRF taking controlled samples from sampling lines does not allow fulfilling continuous on-line control of current values of activity and radionuclide composition of gas aerosol emissions. The method is not rather operative.

In accordance with new standards [1] there is a necessity to develop operative automated control system of emissions allowing as soon as possible recording, computing and analyzing increase of activity of gaseous RAW. Practical value of the results of operative gamma spectrometric analysis is conditioned by a possibility of determining the connection of the results of detecting radionuclide composition and activity of radionuclides with technological processes occurring at nuclear power plant (NPP). Awareness of these connections allows determining the source of emission and taking measures on optimization of technological processes (for example, preventing development of emergencies connected with integrity of pipe lines of reactor primary coolant circuits and controlling hermiticity of PRF fuel element blanket).

The aim of the work was to develop the automated information-measuring complex for on-line continuous control of activity and nuclide composition of emissions